Practical 1: Continental Geotherm and Heat Transfer

NB: Individual practical reports are due next week and will be collected at the beginning of the practical class. Should you hand-in your report later, please do not forget to write on the front page the date at which the report was handed in. In the absence of this information we will have to use the date at which the report was collected. We request a report, not an answer sheet, so please give enough details (including a paragraph on what are the aims of the exercises) so the assessor can understand how you arrived to your solution. Hand written reports are acceptable but please write clearly. Note that 25% of the total mark is allocated to the explanation provided and the neatness of the report.

Non-advanced report: Exercises 1 to 3; Advanced Report: Exercises 1 to 4.

Purpose: To determine the steady state continental geotherm from the 1D conduction-advection heat transfer differential equation and to derive new knowledge on the meaning of surface heat flow.

Outcomes: To develop a deeper understanding of heat transfer in the lithosphere.*Generic skills:* Problem solving ability, computational skills, analytical skills (i.e. finding the math necessary to solve a problem).

Assumed background knowledge: Basic knowledge on heat transfer (check out your lecture notes) and year 12 Mathematics.

Tools that will make your life easier: Matlab, Mathematica, Excel, LiveMath, calculator...

Reading: Turcotte & Schubert: Geodynamics

Some fundamentals: cf lecture notes

Exercise 1 (12 marks): The steady stade geotherm derives from the 1D conduction-advection heat transfert equation (see lecture notes) and knowledge of two boundary conditions. We have seen that with $T=T_0$ at z=0, and with a surface heat flow Q_0 at z=0 the crustal geotherm is:

$$T(z) = -\frac{A}{2k}z^2 + \frac{Q_0}{k}z + T_0 \qquad (1)$$

Determine the crustal geotherm for the following boundary conditions: first boundary condition: $T=T_0$ at z=0; second boundary condition: the heat flow entering the crust is $-Q_m$ at $z=z_c$. Where z_c is the depth of the base of the continental crust assuming that the top of the crust is at z=0.

Exercise 2 (5 marks): Compare your result with equation (1) and demonstrate that the surface heat flow Q_0 is a simple function of Q_m, A, and z_c. Give this function.

Exercise 3 (8 marks): A continental crust is z_c =35 km thick, it has a depth-independent radiogenic heat production A=0.7.10^{-6} W.m^{-3}, a basal heat flow Q_m= 20 10^{-3} W.m^{-2}, a thermal conductivity k=2.5 W.m^{-1}. C^{-1} and a temperature at the surface T_0=20º C?

a/ What is the temperature at the Moho (2 marks)
b/ What is the surface heat flow Q_0? (2 marks)
c/ Assuming that the base of the lithosphere corresponds to the isotherm 1300ºC what is thickness of the lithosphere? (4 marks)

Exercise 4- Continental Geotherm Through Time (Advanced students): Radiogenic isotopes decay through time with a constant time scale λ called the decay constant. Therefore in the past, continental geotherms were warmer as continental crusts were enriched in radiogenic isotopes. ^{40}K, ^{238}U and ^{235}U, and ^{232}Th are the main heat producing radiogenic isotopes. Their concentrations vary through time according to the following relationships where *t*, the time, is in years, and K_0, Th_0 and U_0 are present day concentrations:

- $K(t)=K_0.Exp[-\lambda_K. t]$: With $\lambda_K=5.543\ 10^{-10}$
- $Th(t)=Th_0.Exp[-\lambda_{Th}. t]$: With $\lambda_{Th}=4.9475\ 10^{-11}$

- $U(t)=U_0.(0.992849.Exp[-\lambda_{238U}. t] + 0.00725.Exp[-\lambda_{235U}. t])$: With $\lambda_{238U}=1.55125\ 10^{-10}$ and $\lambda_{235U}=9.8485\ 10^{-10}$

The rate of heat release varies from isotopes to the next. For K, Th and U the rate of heat release is (in $W.kg^{-1}$):

- $H_K = 3.48\ 10^{-9}$
- $H_{Th} = 2.64\ 10^{-5}$
- $H_{238U} = 9.46\ 10^{-5}$
- $H_{235U} = 5.69\ 10^{-4}$

The average content in K, Th and U (in part per million) of present day Archaean crust is:

- K_0=7500 ppm
- Th_0=2.9 ppm
- U_0= 0.75 ppm

1/ Assuming depth-independent radiogenic heat production in the crust and no radiogenic heat production in the mantle, construct the continental geotherm for a 40 km thick crust with an average density of 2700 $kg.m^{-3}$, a basal mantle heat flow of $15.10^{-3}\ W.m^{-2}$ (we assume that the basal heat flow remains constant through time) and a constant temperature at the surface of 20 ºC. The thermal conductivity in the crust and the mantle is: $k=2.5\ W.m^{-1}$.

2/ Determine the temperature at the base of the crust, and the thickness of the continental lithosphere at 4, 3, 2, 1 Ga and present.