GEOS 3101-3801

Practical 3 - Tectonic forces

1 Slab pull and viscosity of the asthenosphere

The aim of this exercise is to estimate the force balance that applies to a subducting ocean plate, with the aim to estimate the viscosity of the mantle. We consider a plate of thickness t subducting with a velocity u_0. The plate has sunk to a depth d in the mantle (Fig. 1) and the extent of the plate parallel to the trench is l.

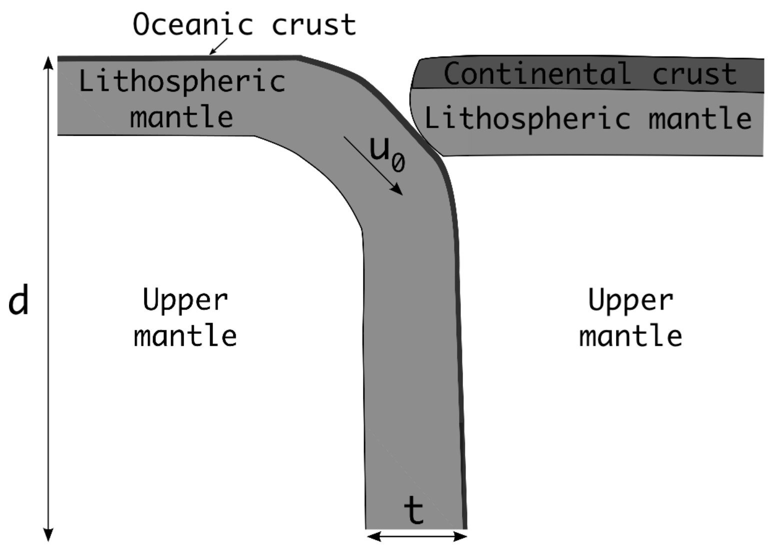

FIG. 1 – Schematic vertical cross-section through a simplified subduction zone.

1. Give values of t, d, l and u_0 based on your knowledge of the Earth.
2. Draw arrows on Fig. 1 to indicate the buoyancy force $\vec{B}$ that applies to the subducting plate and the friction force $\vec{F}$ between the subducting plate and the surrounding mantle. This friction force is the cause of most earthquakes at subduction zones.
3. Express the buoyancy of the subducting plate <u>per unit length</u> as a function of the density difference between the mantle and the oceanic lithosphere $\Delta\rho$. Propose a value of $\left|\left|\vec{B}\right|\right|$ for $\Delta\rho = 60$ kg m^{-3}.
4. *The <u>total</u> friction force parallel to the trench is*

$$\left|\left|\vec{F_T}\right|\right| = 2\,\mu\,u_0\,l\ ,$$

where μ is the viscosity of the mantle and u_0 is the average velocity of subducting plates.

Give the unit of μ in the international system (1 Pa = 1 N m^{-2}).

5. Give an expression for μ assuming equilibrium between the forces acting on the subducting plate *per unit length*. The friction force *per unit length* is

$$\left|\left|\vec{F}\right|\right| = \frac{\left|\left|\vec{F_T}\right|\right|}{l} \ .$$

Does this force balance correspond to weak or to strong coupling between the mantle and the subducting plate ? Propose a value of μ. Is this value a lower or a greater bound for μ ?

2 Ridge push calculated in the half-space cooling model

The aim of this problem is to combine notions of heat diffusion and isostasy in order to estimate the depth of the ocean and the ridge-push as a function of the age of the ocean floor.

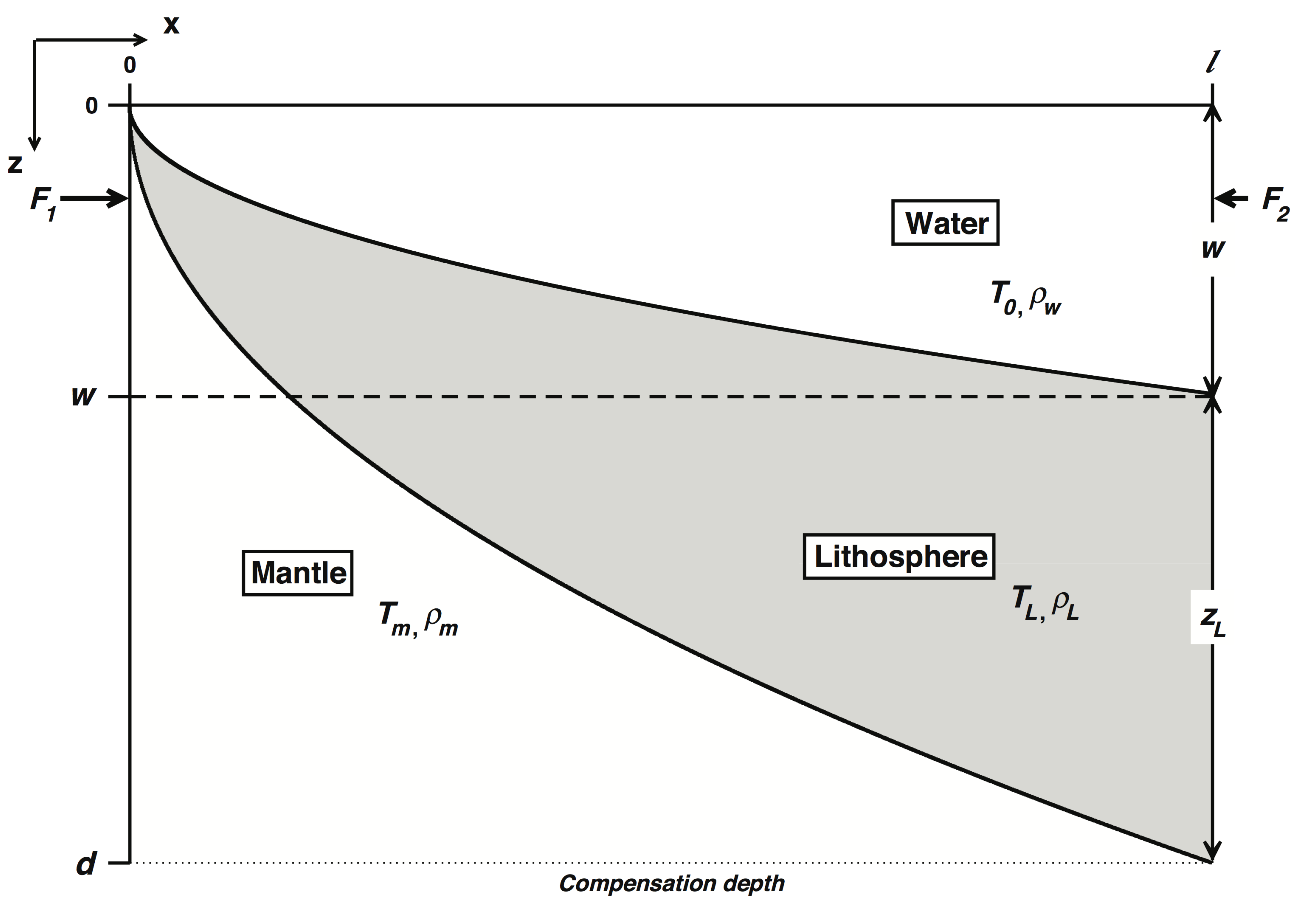

FIG. 2 – Bathymetry of the ocean floor due to the cooling of the lithosphere (not to scale).

1. Write the literal expression of isostatic equilibrium between the mid-oceanic ridge and the oceanic plate at a distance l from the ridge at the compensation depth d, using densities, the lithospheric thickness Z_L and the water depth w.
2. Re-write the expression you obtained in 1 using $(\rho_m - \rho_L)\, Z_L = \int_0^{Z_L} (\rho_m - \rho_L)\, dz$.
3. *The change in density of the oceanic lithosphere with temperature is given by*

$$\Delta\rho = -\rho_m\, \alpha\, \Delta T, \qquad (1)$$

where $\Delta\rho = \rho_m - \rho_L$, $\Delta T = T_m - T_L$, α *is the thermal expansion and* ρ_m *is the reference density.*

Considering cooling in one direction extending to infinity ("half-space cooling model"), the temperature profile of the oceanic lithosphere can be written as

$$\frac{T_m - T_L}{T_m} = \mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa x / u_0}}\right), \qquad (2)$$

where erfc is the complementary error function, κ *is the thermal diffusivity of the lithosphere and* u_0 *is the spreading rate of the oceanic plate.*

   - Combine equations 1 and 2 to express the density difference between the oceanic lithosphere and the mantle.
   - Use this result in the expression you wrote in question 2.

4. *One of the properties of the complementary error function is that*

$$\int_0^{\infty} \mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa x / u_0}}\right) dz = 2\sqrt{\frac{\kappa x}{\pi u_0}}\ . \qquad (3)$$

In the half-space cooling model, because $\rho \rightarrow \rho_m$ *and* $T \rightarrow T_m$ *at the base of the lithosphere, the upper limit of the integral can be changed from* ∞ *to* Z_L*, so that*

$$\int_0^{Z_L} \mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa x / u_0}}\right) dz = 2\sqrt{\frac{\kappa x}{\pi u_0}}\ . \qquad (4)$$

   - Use the given property of the complementary error function (Eq. 4) to express the water depth w as a function of the age of the ocean floor from the expression you wrote in question 3.
   - Use the values given in Table 1 to calculate the depth of ocean floor 80 million years old.

5. Using the concept of gravitational potential energy, express the norm of the force $\left\|\vec{F}\right\|$ resulting from forces $\vec{F_1}$ and $\vec{F_2}$ between the mid-oceanic ridge and the oceanic plate at a distance l from the ridge, down to depth w.

6. Calculate the norm of the force $\left|\left|\vec{F}\right|\right|$ for ocean floor 80 million years old. You will need to use the water depth that you calculated in question 4.
7. <u>*Advanced students :*</u>

   The full expression of the ridge-push is given by

$$\left|\left|\vec{F_R}\right|\right| = \left|\left|\vec{F}\right|\right| + g\rho_m\alpha(T_m - T_0)\kappa t, \quad (5)$$

   where t is the age of the ocean floor.

   Calculate the ridge-push for ocean floor 100 million years old using the values given in Table 1.

   Compare your result to the one you obtained for question 3 of the previous exercise, and comment.

TAB. 1 – Values of the parameters used in the problem.

Parameter	Meaning	Value	Unit (model)
T_m	temperature of the upper mantle	1330	°C
T_0	temperature of the deep ocean	0	°C
g	acceleration of the gravity field	9.81	$m\ s^{-2}$
α	thermal expansion	3×10^{-5}	K^{-1}
κ	thermal diffusivity	1×10^{-6}	$m^2\ s^{-1}$
ρ_m	density of the mantle	3340	$kg\ m^{-3}$
ρ_w	density of the ocean water	1030	$kg\ m^{-3}$